

Analysis of Data Transmission using one modified neural networks

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ABSTRACT

The traditional neural networks cannot provide modern mapping capability which is most important for analysis of data transmission nowadays. Therefore, Sigma-Pi-Sigma neural networks (SPSNNs) are good tool for this operation because of easy architecture. Application of integrated learning approach for neural networks, which uses sigma-pi-sigma neurons, helps us to complete their task for small period of time. It's very necessary for neurons to find the solution of the problem. A final result of our results can be used in order to find the routes of "safety", which we can indicate by position and state of cable lines or ties. For correct analysis and accurate results, we use pulse refractory method with using special device in order to get waveform, which introduce the connection problem. So, Sigma-Pi-Sigma neural network model is used for exact interpretation of altering probe signal. It is crucial that we also used rectangular methods of summation of Fourier series firstly. Therefore, it is main novelty of our investigation.

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1. Introduction

Constructing routes in communication networks, the implementation of switching technology for improving network capacity are the main methods to support the work of high-performance applications. Initial data for algorithms contained in the information about the transmission medium in which the exchange of information is organized. To quickly change the algorithms and to ensure smooth operation of the entire network, a method of continuous information in the computer system of communication channels is the most useful for us. In such way to serve their cable lines sensing and subsequent evaluation is the consequent from the processes described before [1]. The data obtained can be used to construct safe routes, the reliability of which is characterized by the state of their cable lines. To analyze the channel impulse applied reflectometry method using expensive devices for obtaining a signal waveform reflected from different cable inhomogeneities. The most tricky part is the interpreting results. To solve the problems of modeling, presence and control, artificial non-neon networks, particularly multilayer perceptions (MLP) and radial basis networks (RBFN), are widely used. Multilayer perceptions are extremely effective as universal approximators. RBFNs are not inferior to their approximating properties, but the low MLP learning rate based on the back propagation of errors limits their application, especially in real-time problems [2]. The main disadvantage of RBFN is the exponential growth in the number of neurons with increasing size of the input signals, the so-called "size curse".

Widely used modified Sigma-Pi-Sigma neural network (MSPSNN) with an adaptive approach is one of the best ways to find a better multinomial solution for some problem. So, we need to start from a complete multinomial with a given order. After that we use a regularization technique in the process of learning to reduce the number of monomial numbers used in the multinomial, and end up with a new SPSNN with the equal number of monomials (= the number of nodes in the Pi-layer) as in Ps. Experiments show that our MSPSNN behaves more accurately than the traditional SPSNN with Ps [3].

Selection algorithm performed randomly with equal probability every 50 iterations (one iteration of all the examples are presented learning sample). NA settings considered algorithm is based on calculation of the optimal ratio of the internal parameters of the neural network, namely, weighting coefficients and parameters of the functions activated by supplying to the input of a network input vector and fixing the obtained values. The value of the measured error caused by the mismatch between the expected and actually received output signals is input to the learning algorithm. Formation of new solutions in the developed algorithm provides training to search space and to format the best for this task, the combination of parameters. An important concept in signal transmission theory is the principle of orthogonality (based on the coefficient of the correlation function) or reception which allows you to select a system of orthogonal functions (carrier signals) that provide the best transmission quality. When the signals are equal, the orthogonality coefficient is maximum value (instantaneous signal strength). However, if the signals are different, then the coefficient of orthogonality is zero.

Optimum reception is widely used in radar when detecting an object. He received the name "friend-friend" and «friend-alien», what helps us to develop and improve model of neural network Sigma-Pi-Sigma [4].

2. Main Part

ANN consists of artificial neurons, each of which is a simplified model of a biological neuron. The function of artificial neuron is to receive signals from many inputs, to process them in a single way, and to transmit the result to many other artificial neurons, i.e. does the same thing as a biological neuron. Biological neurons are interconnected by axons, the joints are called synapses. In synapses, an amplification or attenuation of an electrochemical signal occurs. Connections between artificial neurons are called synaptic, or simply synapses [5]. The synapse has one parameter – the weight coefficient, depending on its value, one or another change in information occurs when it is transmitted from one neuron to another. It's possible because of the input information is processed and converted into a result, and the training of a neural network is based on the experimental selection of such a weight coefficient for each synapse, which leads to the desired result. Due to the fact that it's necessary for us to maximize accuracy of Sigma-Pi-Sigma neural networks, we are going to use rectangular methods of conversion of Fourier series. It is vital that the rectangular methods are used. These methods are more useful for us because common used, triangular methods [6], lack of 1 dimension, can be indicated by our transmission radar as the error. Therefore it will cause additional costs and system failure, what breaks the rules of using Sigma-Pi-Sigma neural network. Taking into consideration everything what was mentioned before, in the Sigma-Pi-Sigma model the most vital is to use rectangular methods of summation of Fourier series [7]-[10].

The essence of the matrix summation of the series of Fourier is that using the given two infinite matrices of numbers $\Lambda = \|\lambda_k^{(n)}\|$ and $M = \|\mu_k^{(n)}\|$, where $n = 0, 1, 2, \dots$ and $k = 0, 1, 2, \dots, n$, each 2π -periodic function $f(x)$ is matched by a polynomial $U_n(f; x; \Lambda; M)$ kind of

$$U_n(f; x; \Lambda; M) = \frac{a_0 \lambda_0^{(n)}}{2} + \sum_{k=1}^{\infty} [\lambda_k^{(n)} (a_k(f) \cos kx + b_k(f) \sin kx) + \mu_k^{(n)} (-b_k(f) \cos kx + a_k(f) \sin kx)] \quad (1)$$

where $a_k(f)$ and $b_k(f)$ are Fourier coefficients of function $f(x)$.

So Λ and M determine one linear method $U(\Lambda, M)$ of summation the Fourier series, or, analogically, sequence of linear polynomial operators U_n what are defined on class 2π summarized functions. So let's take into consideration $\lambda_0^{(n)} \equiv 1$, $\mu_k^{(n)} \equiv 0$ and write

$$U_n(f; x; \Lambda; 0) = U_n(f; x; \Lambda); \quad U(\Lambda; 0) = U(\Lambda) \quad (2)$$

Now let's turn to some specific linear methods for the matrix summation of Fourier series.

One of the simplest but widely used linear methods of matrix summation is the method of partial sums $S_n(f; x)$ of Fourier series. It can be obtained if the (4) put $\lambda_k^{(n)} \equiv 1$ and $\mu_k^{(n)} \equiv 0$. In this case, we obtain that $U_n(f; x; \Lambda) = S_n(f; x)$. Therefore, for this we put in the formula (3) the value of the coefficients $a_k(f)$ and $b_k(f)$, as a result:

$$\begin{aligned} S_n(f; x) &= \frac{1}{2\pi} \int_{-n}^n f(t) dt + \sum_{k=1}^n \left(\left(\frac{1}{\pi} \int_{-\pi}^{\pi} f(t) dt \cos k t dt \right) \cos k x + \right. \\ &\quad \left. + \left(\frac{1}{\pi} \int_{-\pi}^{\pi} f(t) dt \sin k t dt \right) \right) = \\ &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \left[\frac{1}{2} + \sum_{k=1}^n (\cos k t \cdot \cos k x + \sin k t \cdot \sin k x) \right] dt \\ &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \left[\frac{1}{2} + \sum_{k=1}^n \cos k (t - x) \right] dt. \end{aligned} \quad (3)$$

Trigonometric polynomial order n

$$D_n(x) = \frac{1}{2} + \sum_{k=1}^n \cos k x = \frac{1}{2} + \cos x + \cos 2x + \dots + \cos n x \quad (4)$$

is called Dirichlet kernel. Dirichlet kernel (4) at the same time is the nucleus of the linear method of partial Fourier sum, and at the same time takes place

$$\begin{aligned} D_n(x) &= \frac{1}{2} + \sum_{k=1}^n \cos k x = \frac{1}{2} + \cos x + \cos 2x + \dots + \cos n x = \\ &= \frac{1}{2 \sin \frac{x}{2}} \left[\sin \frac{x}{2} + \sum_{k=1}^n 2 \sin \frac{x}{2} \cos k x \right] = \\ &= \frac{1}{2 \sin \frac{x}{2}} \left[\sin \left(k + \frac{1}{2} \right) x - \sin \left(k - \frac{1}{2} \right) x \right] = \\ &= \frac{\sin \left(n + \frac{1}{2} \right) x}{2 \sin \frac{x}{2}}. \end{aligned} \quad (5)$$

From the last ratio it follows that the nucleus of a partial Fourier's sum is a partial three-metric polynomial of the order, and for it takes place

$$\frac{1}{\pi} \int_{-\pi}^{\pi} D_n(x) dx = 1. \quad (6)$$

1. The second for use and importance in applied mathematics is a matrix linear method of Arithmetic mean $\sigma_n(f; x)$ – the Fejer method. It's obvious that in the (1) we consider $\lambda_k^{(n)} = 1 - \frac{k}{n}$ and $\mu_k^{(n)} \equiv 0$. Therefore,

$$U_n(f; x; \Lambda) = \sigma_n(f; x) = \frac{1}{n} \sum_{k=0}^{n-1} S_k(f; x). \quad (7)$$

Thus, the kernel of the Fejer is the arithmetic mean of the n first Dirichlet kernels, and it means that there is a trigonometric polynomial power of $n-1$. We obtain

$$\begin{aligned} F_n(x) &= \frac{1}{n} [D_0(x) + D_1(x) + \dots + D_{n-1}(x)] = \frac{1}{n} \sum_{k=0}^{n-1} D_k(x) = \\ &= \frac{1}{2n \sin \frac{x}{2}} \left[\sin \frac{x}{2} + \sin \frac{3x}{2} + \dots + \sin \frac{2n-1}{2} x \right] = \\ &= \frac{1}{4n \sin^2 \frac{x}{2}} [(1 - \cos x) + (\cos x - \cos 2x) + \dots + \\ &\quad + (\cos(n-1)x - \cos nx)] = \frac{1 - \cos nx}{4n \sin^2 \frac{x}{2}} = \frac{\sin^2 \frac{nx}{2}}{2n \sin^2 \frac{x}{2}}. \end{aligned} \quad (8)$$

At the same time we consider the equality:

$$\begin{aligned} F_n(x) &= \frac{1}{n} \sum_{k=0}^{n-1} D_k(x) = \frac{1}{n} \left[\frac{1}{2} + \left(\frac{1}{2} + \cos x \right) + \dots + \right. \\ &\quad \left. + \left(\frac{1}{2} + \cos x + \dots + \cos(n-1)x \right) \right] = \frac{1}{2} + \left(1 - \frac{1}{n} \right) \cos x + \\ &\quad + \left(1 - \frac{2}{n} \right) \cos 2x + \dots + \left(1 - \frac{n-1}{n} \right) \cos(n-1)x. \end{aligned} \quad (9)$$

Taking into account what we previously said, we assume that the kernel of the linear Fejer method $F_n(x)$ is a multiple inalienable trigonometric polynomial of power $n-1$ and it's obvious that

$$\frac{1}{\pi} \int_{-\pi}^{\pi} F(x) dx = 1$$

2. A generalization of the above Fejer linear method is the so-called Zygmund linear matrix method. It can be obtained by putting in (4) $\lambda_k^{(n)} = 1 - \left(\frac{k}{n}\right)^s$, $k = 0, 1, \dots, n-1$, $s > 0$, and $\mu_k^{(n)} \equiv 0$. So from (4) we obtain that

$$\begin{aligned} U_n(f; x; \Lambda) &= Z_n^{(s)}(f; x) = \\ &= \frac{a_0}{2} + \sum_{k=1}^{n-1} \left(1 - \left(\frac{k}{n}\right)^s \right) (a_k \cos kx + b_k \sin kx). \end{aligned} \quad (10)$$

Polynomials $Z_n^{(s)}(f; x)$ are called Zygmund sums. When $s=1$ the sums $Z_n^{(s)}(f; x)$ coincide with the sums of Fejer $\sigma_n(f; x)$.

3. We obtain the Vallee-Poussin matrix linear method by considering (4)

$$\lambda_k^{(n)} = \begin{cases} 1, & \omega \varepsilon \nu k = 0, 1, \dots, n-p; \\ 1 - \frac{k-n+p}{p+1}, & \omega \varepsilon \nu k = n-p+1, \dots, n. \end{cases} \quad (11)$$

and $\mu_k^{(n)} \equiv 0$. Therefore,

$$U_n(f; x; \Lambda) = V_{n-p+1}^n(f; x) = \frac{1}{p+1} \sum_{k=n-p}^n S_k(f; x) \quad (12)$$

The kernel of the Vallee-Poussin matrix linear method is the arithmetic mean Dirichlet kernels from $(n-p)$ to n and therefore the relation between the kernels of the Valle-Poussin and Dirichlet is $V_n^{n+1} = D_n(x)$, and, subsequently, between the Valle-Poussin and Fejer kernels it is true that

$$V_0^n = F_n(x) \quad (13)$$

4. If in (4) we consider $\lambda_k^{(n)} = \cos \frac{k\pi}{2n}$, $k = 0, 1, \dots, n$ and $\mu_k^{(n)} \equiv 0$, we obtain the Rogozinski linear matrix method. So $U_n(f; x; \Lambda) = R_n(f; x) = \frac{1}{2} [S_n(f; x + \frac{\pi}{2n}) + S_n(f; x - \frac{\pi}{2n})]$.

Polynomials $R_n(f; x)$ are called Rogozinski sums.

Studies concerning matrix linear summation methods cover a large area that has a number of problem formulations. Each of the above matrix linear methods is an important object in applied mathematics due to their widely application in data transmission. Their various properties were being studied by mathematicians over a long period, but the properties of the little-studied matrix linear methods, which are given by rectangular matrices [11]-[13], have always remained the focus of attention.

For complete analysis we need take into consideration next Theorem.

For even coincidence of sequence of polynomials $U_n(f; x; \Lambda)$ to $f(x)$ over the entire space C it is necessary and sufficient to comply with conditions:

$$a) \lim_{n \rightarrow \infty} \lambda_k^{(n)} = 1$$

b) Sequence of Lebesgue constants is limited

$$L_n(\Lambda) = O(1), n \rightarrow \infty, \quad (14)$$

$$\text{where } L_n(\Lambda) = \sup_{|f| < 1} \|U_n(f; x; \Lambda)\|_C = \frac{2}{\pi} \int_0^\pi |U_n(t; \Lambda)| dt.$$

Let further $\Lambda = \{\lambda_\delta(k)\}$ will be plural of functions, which depend on $k = 0, 1, \dots, n$ and on δ parameter, which alters over the some plural $E_\Lambda \subseteq R$, which has at least limit point. Let also $\lambda_\delta(0) = 1, \forall \delta \in E_\Lambda$. It is worth to mention that in case when $\delta = n, n \in N$, numbers $\lambda_\delta(k) =: \lambda_{n,k}$ are elements of rectangular matrix $\Lambda = \{\lambda_{n,k}\}$ ($n, k = 0, 1, \dots; \lambda_{n,0} = 1, n \in N \setminus \{0\}$) or when $\lambda_{n,k} \equiv 0, k > n$ they would be elements of triangular matrix. Using Λ plural we will put in line every summative function $f(x)$ together with its Fourier series

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx) \quad (15)$$

with

$$\frac{a_0}{2} \lambda_\delta(0) + \sum_{k=1}^{\infty} \lambda_\delta(k) (a_k \cos kx + b_k \sin kx), \delta \in E_\Lambda. \quad (16)$$

If this series with each $\delta \in E_\Lambda$ are Fourier series of some function, we will mark it as $U_\delta(f; x; \Lambda)$, and when $\delta = n, n \in N$ as $U_n(f; x; \Lambda)$. Therefore, every plural Λ specifies a method for constructing operators $U_\delta(f; x; \Lambda)$. We can say the same thing is happen with a multiplier Λ by visual specific method of summation of Fourier series.

If the sequence $\{\lambda_\delta(k)\}_{k=0, \infty}$ makes series

$$\frac{1}{2} + \sum_{k=1}^{\infty} \lambda_\delta(k) \cos kt \quad (17)$$

Fourier series of some summative function, then, analogically, we have the equality

$$U_\delta(f; x; \Lambda) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x+t) K_\delta(t; \Lambda) dt, \quad (18)$$

where $K_{\delta}(t; \Lambda) = \frac{1}{2} + \sum_{k=1}^{\infty} \lambda_{\delta}(k) \cos kt$.

3. Conclusions

This investigation provides us with methods of analysis of calculation Sigma-Pi-Sigma corrections. The scientific novelty of the developed architecture of the artificial neural network lies in the successful combination of the advantages of radial basis and sigmoidal activation functions. The gradient learning algorithm allows you to adjust the synaptic weights of the network in real time with a given accuracy. The high learning speed and universal approximating properties of the proposed network are of practical importance; they will be especially useful when processing multidimensional functions of a vector argument. In the future, research will include the development of a sigma-pi network without using the direct product procedure of the input vectors of the hidden layer.

The operators (18) define rectangular summation methods for Fourier series [14]-[20] whose efficiency for the study of information transmission compared to triangular methods (6) is $\ln n$ times higher, what increases the probability of reliable data transmission dramatically [21] and the Sigma-Pi-Sigma neuron interconnection.

At the same time, we used firstly rectangular methods of matrix summation of Fourier series what hadn't been represented before.

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